Department of Mathematics
Sardar Vallabhbhai National Institute of Technology

$\sqrt{A^2}$ MaThing

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" The book of nature is written in the language of mathematics. "

– Galileo Galilei

Welcome Message

I am overwhelmed to release this issue of AMaThing commemorating the occasion of the International Day of Mathematics (IDM) celebrated on March 14. Mathematics is crucial in our daily activities, which we often underestimate. International Mathematical Union (IMU) took IDM as an initiative to promote mathematics in a non-mathematical world.

Moreover, in recent years, the public has seen a world explode with unscientific truth and flawless statements due to the lack of communication between academia and science. Communicating science is integral for a researcher and vital in upholding truths. Our motive for releasing this issue is to help society understand various advances in research in mathematics and their allied areas, assisting the stakeholders in learning new approaches.

I thank the entire team for their painful efforts in producing this magazine. Over the years, we have raised the magazine's standards, and it is the best copy produced.

Happy Reading!

Dr. Jayesh M. Dhodiya
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Exploring Advanced Theoretical and Computational Methods for Fluid Flow in Porous Media

Nagesh Sahu

Introduction

Understanding the movement of fluids through porous materials is a key area of research in fields like hydrogeology, petroleum engineering, and environmental science. These systems are inherently complex, often requiring nonlinear partial differential equations (PDEs) to model the interactions between fluid dynamics, porosity, and transport processes. This article dives into the core equations that describe flow in porous media, discusses analytical and computational techniques for solving them, and evaluates modern numerical methods to simulate these phenomena accurately.

Mathematical Models of Fluid Flow in Porous Media

At the heart of porous media flow lies Darcy's Law, which relates the fluid flux to the pressure gradient, permeability, and fluid viscosity. While Darcy's Law works well for slow and viscous flows, real-world scenarios often require more nuanced models. For example, Brinkman's equation adds a term to account for viscous shear stresses, while Forchheimer's equation introduces an inertial component to address high-velocity flows. These extensions are crucial for capturing behaviours like turbulence or non-Newtonian fluid dynamics in complex porous structures.

Analytical and Semi-Analytical Solutions

Despite their complexity, analytical approaches remain vital for validating numerical models and uncovering theoretical insights. These are a few key methods:

- **Perturbation Techniques**: This approach is useful for systems with mild nonlinearities. These methods approximate solutions by expanding around a small parameter (e.g., low flow rates).
- **Integral Transforms**: Laplace or Fourier transforms simplify linear transient flow problems by converting PDEs into algebraic equations.
- **Homotopy Analysis Method (HAM)**: This flexible approach generates convergent series solutions for strongly nonlinear problems without relying on simplified assumptions.
- **Adomian Decomposition Method (ADM)**: This method breaks down nonlinear equations into simpler components, enabling iterative solutions without discretization.

These techniques provide benchmark results and reveal the influence of parameters (viz. permeability or viscosity) in a flow behaviour.

Computational Approaches for Nonlinear PDEs

Generally, analytical methods are not reliant on heterogeneous or highly nonlinear systems. In such scenarios, computational strategies play an important role. Some popular methods include:

- **Finite Difference Method (FDM)**: This method is ideal for structured grids and straightforward geometries. FDM approximates derivatives using grid-based discretization.
- **Finite Element Method (FEM)**: In this method, we excel an irregular domain (e.g., fractured rock) by dividing the domain into smaller and adaptable elements.
- **Finite Volume Method (FVM)**: This method ensures mass and momentum conservation at discrete volumes, making it a viable approach for fluid flow simulations.
- **Lattice Boltzmann Method (LBM)**: This model is implemented when models flow at a mesoscopic scale using particle interactions,

which is particularly effective for multiphase flows or complex geometries.

An efficient method in choosing a correct approach (technique) depends on various factors, including computational resources, required accuracy, the system's geometric complexity, etc.

Applications and Cutting-Edge Developments

Research in porous media flow has transformative applications. A few of them are listed as follows:

- **Groundwater Management**: We can improve aquifer recharge and contaminant transport models.
- **Oil and Gas Recovery**: It is possible to optimize reservoir simulations to boost hydrocarbon extraction or CO_2 storage.
- **Environmental Cleanup**: We can develop strategies for soil decontamination or filtering pollutants.
- **Biomedical Engineering**: An emerging approach is studying drug delivery through tissues or designing artificial organs.

Recent advancements leverage high-performance computing (HPC) to handle large-scale simulations while machine learning accelerates model calibration. Hybrid methods that blend analytical and numerical techniques are gaining momentum, offering faster solutions without sacrificing precision.

Conclusion

Modeling fluid flow in porous media is a multifaceted challenge that bridges theory and computation. By combining analytical insights with advanced numerical tools, researchers can tackle real-world problems with various applications, from managing groundwater resources to designing medical treatments. As computational systems become more efficient and newer methods evolve, our ability to predict and control these systems will deepen and drive innovation across science and engineering.

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The Transportation Problem: An Overview

Hartik Bapoliya, and Indira Tripathi

Background

The transportation problem is a fundamental optimization problem in logistics and operations research. It entails determining the best cost-effective way to transport items from various locations, like factories or warehouses, to different places, such as retail stores or distribution hubs. The primary motive is to meet supply and demand restrictions while minimizing transportation costs.

This problem occurs in many sectors like manufacturing, supply chain management, and logistics, where efficient distribution of products keeps the overall cost low and improves service levels.

History

For over a century, researchers have been studying the transportation problems. It was initially defined in the early 20th century by Frank L. Hitchcock in 1941 [2] and further enhanced by T. C. Koopmans [3] in 1949. Koopmans and Leonid Kantorovich made significant contributions to the field by developing mathematical techniques for optimal resource distribution, an achievement that earned them the Nobel Prize in Economics in 1975.

The transportation problem was formulated using linear programming as a well-defined optimization model. This was further improved by George Dantzig's [1] simplex method and specialized algorithms like the MODI (Modified Distribution) method and the stepping-stone method.

Model Formulation

Generally, transportation problems are expressed in linear programming terms. Consider the following components:

- **Sources:** Locations that provide supply goods, denoted as r different sources.
- **Destinations:** Locations that need goods, denoted as t different destinations.
- **Supply constraints:** Supply capacity of every source has a fixed capacity.
- **Demand constraints:** Requirements for every destination must be fulfilled.
- **Transportation cost:** Transporting goods from a source location to a destination.

The transportation problem can be expressed as:

$$\text{minimize} \quad \sum_{s=1}^{r}\sum_{d=1}^{t} e_{sd}x_{sd}$$

Subject to

$$\sum_{d=1}^{t} x_{sd} \leq a_s, \; \forall \; s = 1, 2, \cdots, r$$
$$\sum_{s=1}^{r} x_{sd} \geq b_d, \; \forall \; d = 1, 2, \cdots, t$$
$$x_{sd} \geq 0, \; \forall \; s, d$$

Where

- The quantity of unit goods which transported from the s^{th} source to d^{th} destination is denoted by x_{sd}.
- The cost per unit of transportation from s^{th} source to d^{th} destination is denoted by e_{sd}.
- The availability of goods at source s is a_{s}.
- The requirement of goods at destination d is b_{d}.

For finding an initial feasible solution, multiple methods like the Northwest Corner method (NWCM), Least Cost Method (LCM), and Vogel's Approximation Method (VAM), and then for the optimal solution techniques like the MODI method are used.

Applications

The transportation problem is widely used in various industries and applications, including:

1. **Supply Chain Management:** For optimizing the distribution of raw materials and finished products to minimize costs.
2. **Logistics and Freight Transport:** In finding the best routes and modes of transportation for goods movement.
3. **Healthcare and Emergency Services:** For efficiently allocating medical supplies and resources to hospitals and clinics.

4. **Energy Distribution:** In managing fuel supply chains for power plants and distribution centers.
5. **E-commerce and Retail:** For optimizing warehouse-to-customer deliveries to ensure timely fulfillment and reduce costs.
6. **Public Transportation Planning:** In determining optimal routes for buses, trains, and other transport modes to maximize efficiency.
7. **Waste Management:** For optimizing the collection and disposal of waste from different locations.

Example

Four petrochemical plants in Malaysia create polymer and sell it to China, the Middle East, Europe, and South East Asia through the trading business "Mitco Labuan." The following table lists each route's supply and demand and unit transportation costs. Thousands of Malaysian Ringgit (MR) represent the unit transportation costs (Table 2.1).

Variations in other distances and exchange rates affect the shipping cost per unit. Therefore, the trading organization needs to allocate production capacities to the different demand destinations most efficiently to decrease the shipping cost.

Conclusion

The transportation problem remains a cornerstone of optimization and logistics and supply chain management decision-making. With advancements in computational techniques and artificial intelligence, modern solutions have evolved to handle large-scale and dynamic transportation networks efficiently. As businesses and economies expand, solving transportation problems efficiently will remain crucial for minimizing costs and improving service efficiency.

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Table 2.1: Unit Cost of Shipping (MR '000)

Plant	China	Middle East	South East Asia	Europe	Capacity of production
P1	200	300	100	600	110
P2	400	350	150	650	75
P3	300	250	150	600	95
P4	500	400	200	700	125
Requirement	200	90	40	45	

On Atangana–Baleanu Fractional Derivative

Shinde Bhavika Dadaso

Abstract. In this article, the evolution and applications of the Atangana-Baleanu fractional derivative have been discussed. This article gives a brief historical background of fractional derivatives, mathematical expression, theoretical advancement, and practical applications.

Introduction

The study of derivatives and integrals of arbitrary orders (specifically non-integer) in fractional calculus is gaining importance because of its effectiveness in modeling systems in various fields such as mathematics, physics, and engineering. Some basic fractional derivatives like the Caputo and Riemann–Liouville derivatives are widely used. However, these fractional derivatives have drawbacks, such as singular kernels and local properties. In real-world phenomena, specifically in systems with non-local interactions and memory effects, these derivatives create problems in accurate detection.

In 2016, Abdon Atangana and Dumitru Baleanu discovered a new fractional derivative called Atangana-Baleanu fractional derivative. It overcomes the restrictions of traditional fractional derivatives and provides its applications in different areas of scientific domains, like differential equations, geometric function theory, etc. The impact of this fractional derivative also occurs in mathematical modeling of critical systems.

Historical Background

The great scientist L'Hôpital came up with an interesting question about the derivative of a function with order 1/2. This question led to the rich history of fractional calculus in 1965. Many scientists like Riemann, Liouville, Caputo, and Grünwald have contributed significantly to the development of fractional derivatives.

These basic fractional derivatives come with crucial limitations, and these limitations motivate the generalization of the derivative called the Atangana-Baleanu fractional derivative. This derivative contains the non-local and non-singular kernel, which was explained by the generalized Mittag-Leffler function.

Due to the steady behavior of this fractional derivative and its applications in a wide range of problems, many researchers came forward to explore its mathematical expression and potential extension to different fields.

Mathematical Expression

A non-singular and non-local kernel, dependent on the generalized Mittag-Leffler function, defines the Atangana-Baleanu fractional derivative. This derivative maintains the memory effect to simulate critical systems by recognizing the whole function history.

The mathematical formulation is given by:

$$^{C}D^{\delta}_{a+}f(t) = \frac{B(\delta)}{1-\delta}\int_a^t E_\delta\left(-\frac{(t-x)^\delta}{1-\delta}\right)f'(x)dx$$

Where:

- δ - Fractional order $(0 < \delta < 1)$,
- E_δ - The Mittag-Leffler function,
- $B(\delta)$ - Normalization constant.

Theoretical Advancements

In the field of fractional derivatives, the Atangana-Baleanu fractional derivative gives a great advantage in theoretical development given as follows:

- With the non-singular and non-local kernel generalizing classical fractional derivative.
- Giving memory effects a more concrete representation via mathematics.
- Supporting the advancement of novel computation methods for fractional differential equation solutions.
- Resolving challenges related to singular kernels in classical derivatives.

Applications

- **Epidemiology**: To better represent disease dynamics in the real world, SEAIR epidemic models add nonlocal interactions and memory effects [2].

- **Geometric Function Theory**: The Atangana–Baleanu fractional integral operator is employed to define new subclasses of analytic functions, leading to advances in geometric function theory, such as distortion theorems and coefficient estimates [1].

- **Differential Equations**: It solves linear and nonlinear fractional differential equations, offering new existence and unique outcomes. These differential equations are solved numerically using methods such as the Chebyshev collocation technique [3].

- **Solute Transport in Porous Media**: The AB derivative models non-Darcy flow in porous media to better understand the solute transport dynamics. [4]

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Machine Learning and Twin Support Vector Machine: An Emerging Approach

Patel Princekumar Dharmendrabhai

Machine learning has become a revolutionary technology, driving innovation in fields such as healthcare, artificial intelligence, robotics, and finance. Fundamentally, it helps the systems to identify patterns and structures among data, that allows to make judgments or predictions without explicit programming. Support Vector Machines (SVM) and their more advanced variant, Twin Support Vector Machines (TWSVM), are very popular for classification tasks among the multiple machine learning approaches. In many cases, TWSVM is the favored option because it offers a more effective approach to solving classification difficulties.

What is Machine Learning?

A subfield of artificial intelligence (AI), machine learning is concerned with creating algorithms that let computers learn from input data and get better at what they do. We must comprehend the three main categories of machine learning before moving on to specific algorithms. These three categories are as follows:

- **Supervised Learning**: It works with labeled datasets and concentrates on learning patterns through the relationship between variables and known outcomes.
- **Unsupervised Learning**: It concentrates on learning patterns and structure between unlabeled datasets.
- **Reinforcement Learning**: The model learns by trying things out, seeing what works, and getting rewarded for good choices or corrected when it makes mistakes, similarly as we learn through experiences.

Support Vector Machine (SVM)

SVM is a machine learning algorithm that categorizes data by finding the best line or hyperplane that separates different classes in datasets. It is very useful for binary classification problems due to its high accuracy and robustness. However, there are some limitations of SVM, such as difficulty in handling imbalanced data and computation being very complex for large datasets.

Mathematically, for a given set of training samples (x_i, y_i), where $x_i \in \mathbb{R}^n$ and $y_i \in \{-1, 1\}$, SVM aims to find a hyperplane $w^T x + b = 0$ that maximizes the margin between the two classes. It can be formulated as the following optimization problem:

$$\min_{w,b} \frac{1}{2}||w||^2 \tag{4.1}$$

subject to:

$$y_i(w^T x_i + b) \geq 1, \ \forall \ i \tag{4.2}$$

Twin Support Vector Machine (TWSVM)

TWSVM solves the limitations of traditional SVM. It is a binary classifier that constructs two non-parallel hyperplanes instead of one. Each hyperplane is associated with one class and as far away from the other as possible.

Mathematically, TWSVM constructs two hyperplanes $w_1^T x + b_1 = 0$ and $w_2^T x + b_2 = 0$, each closer to one class. The optimization problems for TWSVM are:

$$\min_{w_1,b_1} \frac{1}{2}||Aw_1 + eb_1||^2 + C_1 e_2^T \eta_2 \tag{4.3}$$

Subject to:

$$-(Bw_1 + e_2 b_1) + \eta_2 \geq e_2, \quad \eta_2 \geq 0, \tag{4.4}$$

and

$$\min_{w_2,b_2} \frac{1}{2}||Bw_2 + eb_2||^2 + C_2 e_1^T \eta_1 \tag{4.5}$$

Subject to:

$$(Aw_2 + e_1 b_2) + \eta_1 \geq e_1, \quad \eta_1 \geq 0, \tag{4.6}$$

where A and B represent the data matrices of the two classes, C_1, C_2 are regularization parameters, η_1 and η_2 are slack variables, and e_1, e_2 are vectors of ones.

Advantages of TWSVM

- **Faster Computation**: TWSVM transforms a single large quadratic programming problem of SVM into two smaller QPPs and reduces the computational time by a quarter of what traditional SVM takes.
- **Better Handling of Imbalanced Data**: Since TWSVM constructs different hyperplanes for each class, it can handle imbalanced data more effectively.
- **Higher Accuracy**: In high-dimensional datasets, TWSVM provides better classification performance.

Applications of TWSVM

TWSVM has been very useful in various domains, including:

Bioinformatics & Medical Diagnosis

- **Bioinformatics**: In identifying protein structures or classifying gene expressions based on genomic data.
- **Disease Prediction**: Using Medical records and diagnostic data, applied to detect diseases like diabetes, COVID-19, and heart disease.
- **Cancer Classification**: Classifying diseases like cancer based on patient data like imaging scans, where rapid diagnosis is crucial. (e.g., lung, breast, and brain cancer)

Financial Market Analysis

- **Stock Market Prediction**: Using historical market data helps forecast stock trends.
- **Credit Risk Assessment**: Used by banks to classify loan applicants based on financial features.

Image Processing

- **Face Recognition**: Used for the identity verification and security system.
- **Object Detection**: Classifying and grouping images for autonomous vehicles and surveillance applications.
- **Handwriting Recognition**: Used for recognizing handwritten text and converting it into digital format.

Text Classification & Natural Language Processing (NLP)

- **Spam Detection**: Used for separating spam emails.
- **Sentiment Analysis**: To analyze customer reviews for products/ shops, tweets, and social media posts.
- **Document Categorization**: Used to group articles into several categories, such as entertainment, sports, and news.

Remote Sensing & Environmental Monitoring

- **Land Cover Classification**: Helps study satellite images, which is useful in agriculture and urban planning.

Speech Recognition & Audio Processing

- **Speaker Identification**: Using audio samples in a voice detection system helps in speaker identification.
- **Music Genre Classification**: Using audio properties helps to classify and sort the songs.

Recent Advances and Research Directions

Recent research in TWSVM has focused on:

- **Feature-Weighted Kernel Methods**: By their importance, assigning different weights to features that help to improve performance (such as Centered Kernel Alignment).
- **Fuzzy TWSVM**: Using fuzzy membership helps improve accuracy and robustness against noisy data.
- **Hybrid Models**: Hybrid models combine deep learning and TWSVM to improve efficiency for complex datasets.
- **Multiple Kernel TWSVM (MK-TWSVM)**: Use multiple kernels simultaneously to improve algorithms.
- **Cost-Sensitive TWSVM**: Modify objective function, which helps to handle class imbalanced datasets with significant skew.

Conclusion

In machine learning out of multiple classification methods, Twin Support Vector Machine has emerged as a significant advancement. Its ability to easily handle large-scale and imbalanced datasets makes it more preferable choice over basic SVM. As research continues, in corporating new kernel functions, hybrid techniques, and optimization methods will further enhance its effectiveness in real-world applications.

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Stability Analysis of Rabbit-Fox Model

Radadiya Hardikkumar

Introduction

Let's consider a population of foxes, denoted as F, and a population of rabbits, denoted as R. These variables represent the number of species at any given time respectively.

Assume that the fox population changes at a rate proportional to its current population. This assumption is trivial. However, for simplicity, we assume that they live forever and have no upper population limit. A basic understanding of calculus gives us insight into how it leads to exponential growth.

On the other hand, the rabbit population grows three times its current population, much faster than the foxes. However, they are also preyed upon. Naturally, their population decreases at a rate proportional to the number of foxes. This interaction makes the system more complex, but we aim to understand how these populations change over time under different initial conditions.

Visualizing the System

Graphically, we can represent the rabbit population on the x-axis and the fox population on the y-axis. Suppose we start with 10 rabbits and 10 foxes. This initial state is represented as the point $(10, 10)$. If we plug these values into the governing differential equations, we obtain their derivatives, which gives us an insight into population change at each moment. The results show that the rabbit population is increasing at twice the rate of the fox population.

By treating these derivatives as velocity components, we can determine the direction of movement in the phase plane. Since both populations are increasing, the point moves up to the right. Even though rabbits are being eaten, their reproduction rate compensates for it.

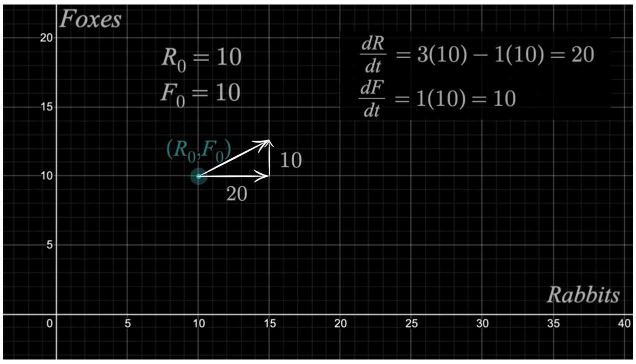

Figure 5.1: Graph of Population of Rabbit and Fox

However, if we start with only two rabbits and ten foxes, the equations indicate that the rabbit population will decline. In this case, there aren't enough rabbits to sustain their population, and they eventually die out. This situation moves the point up and to the left, ultimately leading to zero rabbits.

Phase Plane and Matrix Representation

If we repeat this process for various initial conditions, we obtain a phase plane, a field of directional arrows demonstrating the population evolution over time. Instead of manually computing each point, we can express the system in matrix form:

$$\begin{bmatrix} R' \\ F' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} R \\ F \end{bmatrix} \tag{5.1}$$

Here, the matrix contains the coefficients that determine the individual species' influences compared with the other's population dynamics. When we multiply this matrix by the population vector, we obtain the rates of change at any given point.

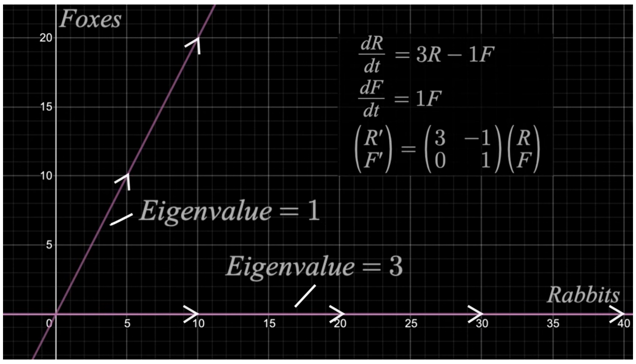

Figure 5.2: Eigen value and Eigen vector of system of ODEs

By analyzing this system, we observe that certain lines (eigenvectors) remain unchanged under transformation. These eigenvectors represent population ratios that remain constant over time. For example, if we start with twice as many foxes as rabbits, the populations will always maintain that ratio.

Stability Analysis

The Jacobian matrix is given by:

$$J = \begin{bmatrix} \alpha - \beta y & -\beta x \\ \delta y & \delta x - \gamma \end{bmatrix} \tag{5.2}$$

At the coexistence equilibrium, the characteristic equation gives eigenvalues:

$$\lambda = \pm i\sqrt{\alpha\gamma} \tag{5.3}$$

Since the eigenvalues are purely imaginary, the system exhibits periodic oscillations.

Equilibrium Points

Setting $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$, we obtain the equilibrium points:

- **Extinction equilibrium:** $(0, 0)$
- **Rabbit-only equilibrium:** $\left(\frac{\gamma}{\delta}, 0\right)$
- **Coexistence equilibrium:** $\left(\frac{\gamma}{\delta}, \frac{\alpha}{\beta}\right)$

Long-Term Behavior

The eigenvectors divide the phase plane into regions with distinct outcomes.

- **Above the top eigenvector**: The rabbit population eventually reaches zero as foxes outcompete them. The system evolves toward the y-axis, meaning the foxes *win*.
- **Below the bottom eigenvector**: Rabbits outpopulate foxes over time. Although they are being hunted, their reproduction rate allows them to dominate.
- **On the eigenvector**: The populations maintain a stable ratio, meaning neither species takes over completely.

Suppose we adjust the system parameters, such as increasing the rabbit reproduction rate, which shifts the eigenvector and expands the region where rabbits dominate. Similarly, increasing the fox reproduction rate shifts the eigenvector downward, favoring the fox population.

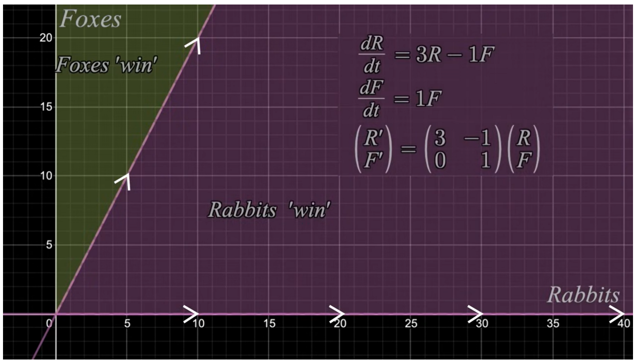

Figure 5.3: Eigenvectors Differentiate Phase plane of survival of Fox and Rabbit

Conclusion

This model provides insight into predator-prey dynamics and how initial conditions determine long-term outcomes. We can predict whether a species will thrive, decline, or coexist in equilibrium by analyzing eigenvectors and phase planes.

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Mathematical Modeling and Optimal Control: A Framework for Intelligent Decision Making

Vishwa Bhatt

An Insight of Mathematical Model and Optimal Control

In the modern era, Mathematical modeling and optimal control are essential for analyzing and improving real and complex systems across different fields. After converting real-world problems into mathematical form, these techniques help optimize decision-making processes, elevate efficiency, and achieve desired outcomes.

From public health to robotics, optimal control strategies are crucial in solving modern problems. This article explores the introduction of mathematical modeling and optimal control and the various applications of mathematical modeling and optimal control in significant areas.

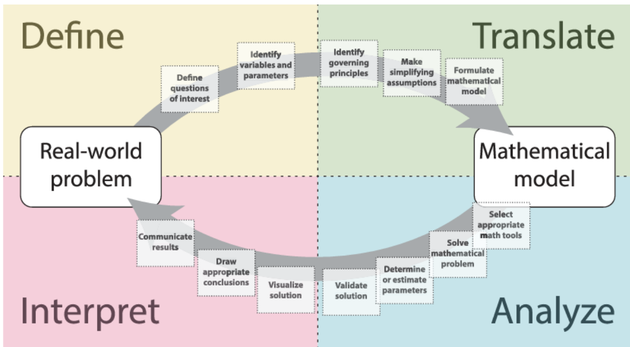

Figure 6.1: Mathematical model process [1].

Mathematical Modeling

Mathematical Modeling uses mathematical structures and equations to represent real-world systems. The purpose of using mathematical modeling is to gain an understanding of the real-world system's behavior and make predictions about its future outcomes. A model can be as simple as a linear equation or as complex as a system of partial differential equations.

Steps of Mathematical Modeling

1. **Define Problem**: We must define a real-world problem and identify important variables and parameters.

2. **Formulation:** We need to develop mathematical equations describing relationships between variables.

3. **Analysis:** Solve the equations using a suitable method, which can be analytic or numerical.

4. **Validation:** Compare the model prediction with real-world data to check accuracy.

Optimal Control

Optimal control is a mathematical optimization method for deriving control policies like minimizing costs, maximizing efficiency, or achieving a desired state in the shortest possible time. Optimal control is widely used in engineering, economics, and scientific fields.

Principles of Optimal Control

1. **Control Variables:** The parameters that can be adjusted to influence the system's behavior (e.g., dosage, temperature).

2. **Objective Function:** Criteria of optimization (e.g., minimize cost, maximize production).

3. **Constraints:** The limitation or requirements that should be satisfied (e.g., safety limits).

4. **Optimization Methods:** Techniques can be used to find the optimal control policy (e.g., Pontryagin's maximum principle, dynamic programming).

Real world Implementation of Mathematical Model and Optimal control

Application in Healthcare

Epidemiology: controlling Disease Spread

In epidemiology, mathematical models such as the SIR (Susceptible-Infected-Recovered), SEIR (Susceptible- Exposed-Infected-Recovered), and SIRS (Susceptible-Infected-Recovered- Susceptible) models help in predicting disease spread. Optimal control strategies like vaccination, quarantine, social distancing, and treatment policies are designed to minimize outbreaks while balancing economic and social factors. The control function applied to these models helps to find the best strategies at different stages of an epidemic like COVID-19.

Medical Treatment: Optimal Drug Administration

In optimizing drug dosage, optimal control plays a crucial role. Mathematical models help design personalized drug regimens for chronic diseases such as diabetes and cancer that minimize side effects and provide effective therapy. Additionally, models assist in planning for the treatment of infectious diseases such as HIV and COVID-19 by minimizing viral load while reducing side effects.

Biological Systems and Homeostasis

Mathematical modeling is used in biological systems to study homeostasis. Optimal control methods help to understand regulatory mechanisms in processes such as insulin regulation in diabetes treatment and maintaining a constant body temperature without the effect of external temperature and blood pressure regulation.

Environmental and Ecological Application

Mitigation of Pollution and Climate change

Optimal control helps in climate change mitigation strategies, managing resources, and reducing carbon releases. Models used in designing policies for energy use, deforestation control, and carbon trading mechanisms. These policies are essential in achieving global climate goals and long-term environmental sustainability.

Ecosystem stability and Wildlife Maintenance

Predator-prey dynamics and species population growth's mathematical model are very important to maintain stability in the ecosystem. Optimal control is used for eco-friendly hunting or harvesting policies, which balance economic gains with stability.

Applications in Engineering and Finance

Financial Markets: Portfolio Control and Financial Management

Mathematical models with stochastic control techniques are used in finance to optimize portfolio allocation, manage risk, and optimize investment strategies with maximum returns and minimum risk. Governments also use models to design policies that maximize economic growth by reducing inflation and unemployment risk.

Robotics and Autonomous System

Robotics depends on control theory for motion planning, navigation, trajectory optimization, and feedback control of autonomous systems. Like autopiloted cars, control strategies guarantee smooth navigation by minimizing energy intake and avoiding obstacles. Artificial intelligence and optimal control elevate automation, consistency, and adaptability in robotics.

Conclusion

Mathematical modeling and Optimal control give a priceless insight into complex systems of various disciplines. These mathematical tools are helpful in all fields, from healthcare to finance and from ecology to robotics, to create strategies that optimize desired outcomes while minimizing costs or risks. The applications of optimal control will continually grow as technology advances, which will benefit the future of science and engineering.

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Frames and Their Use in Signal Processing

Sanjul Mishra

Introduction

We face poor internet connectivity on various occasions, leading to data and information loss. To overcome this issue, mathematicians and computer scientists have devised an idea of **frames**. A frame is an organized way to handle data with a possibility of recovery even when data is lost during communication. Bodmann and Paulsen introduced the mathematical description of frames [1].

We would like to show the idea through a small example. Let's take an image, divide it into tiny pieces, and lose a few pieces. We can still reconstruct the original image, which contains fewer pixels than the original picture. The hidden information behind the lost data transmission motivates us to use frames. A frame is a unique set of data points that assist us in retrieving lost information while reconstructing the original picture. One such classification is a "Two-uniform frame." This frame tries to keep the error, i.e., information between the reconstructed and original messages, as small as possible.

Frames aren't just random collections of data; they follow structured patterns, and these patterns can be represented using graphs, which are mathematical structures of a collection of points connected by lines. These graphs help us determine the frame's practicality in recovering lost information. Some unique graphs, like Hadamard and Conference Matrices, can be used to create optimal frames. When such matrices are used, they ensure that the reconstructed message contains minimal error even though the original message was significantly lost.

Another class of frames is known as "Equal-norm frame." These ensure that even if there is a considerable loss in the data, the data can still be reconstructed with the minimum possible error. These frames are widely used in communication networks to improve reliability.

In our present digital era, we expect bufferless online streaming and sharing of secure information among space missions, which boils down to errorless data transmission. Scientists are discovering new ways to protect lossless data transmission by studying frames and graphs. As technology advances, new types of frames are being developed to handle even more erasures. Further, scientists are exploring novel approaches to utilize frames in emerging quantum computing and artificial intelligence fields. The future of secure and reliable data transmission depends on these exciting mathematical discoveries.

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Fractional Differential Equations: A Journey from Historical Insights to Modern Day Applications

Vinchhi Foram Dhanji

Initial Developments

As the apple fell on Isaac Newton was the beginning of gravity. A similar thought about the order of derivatives sparked in L'hopital's mind: "What if the order will be $\frac{1}{2}$?"

In the 16th century, L'hopital wrote a letter to Leibniz about fractional order differentiation. The idea of fractional derivatives has been in the spotlight since the 18th century, as shown by the works of Leonhard Euler and Joseph Fourier. Euler analyzed the concept of fractional calculus as it applies to series expansion and special functions. His work made further progress in developing the theory of fractional differentiation.

In 1819, S. F. Lacroix first mentioned the arbitrary order derivative in his book. In 1823, Niels Henrik Abel and Carl Gustav Jacob Jacobi gave the concept of generalizing the derivative to non-integer order, but it was not fully defined at this stage. In 1830, the Riemann-Liouville fractional integral and the Liouville fractional derivative offered a more structured mathematical approach to fractional calculus. Still, it was only in theory, and it lacked applications. During the 19th century, mathematical interest in fractional calculus grew on and off, but it was still somewhat explored in Mechanics and Physics. In the early 20th century, many researchers pushed the development of more systematic theories for fractional derivatives.

Paul Lévy, in his 1925 work "Calcul des Probabilités," applied fractional calculus to probability theory. The 1920s is often regarded as the best time for implementing fractional derivatives in Engineering, Control theory, and Physics driven by work in thermodynamics, viscoelasticity, and diffusion processes. It expanded to fields of signal processing and dynamical systems. It is widely used in physics, finance, and even biological systems. It is used for model systems by numerical methods that exhibit memory and hereditary properties such as viscoelastic materials, anomalous diffusion, and fractal geometry. Now, it has become a cutting-edge research area.

Few Important Fractional Derivatives

- Liouville derivative [2]:

$$D^{\gamma}\left[f(x)\right] = \frac{1}{\Gamma(1-\gamma)}\frac{d}{dx}\int_{-\infty}^{x}(x-\xi)^{-\gamma}f(\xi)\,dx, \quad -\infty < x < +\infty$$

- Riemann-Liouville fractional integral of f of order α [2]:

$$D^{\gamma}f(x) = \frac{1}{\Gamma(n-\gamma)}\frac{d^n}{dx^n}\int_{a}^{x}\frac{f(t)}{(x-t)^{\gamma-n+1}}dt, \quad n-1<\gamma<n$$

- Cuputo derivative [2]:

$$^{C}D^{\gamma}f(x) = \frac{1}{\Gamma(n-\gamma)}\int_{0}^{x}\frac{f^{(n)}(t)}{(x-t)^{\gamma-n+1}}dt, \quad n-1<\gamma<n$$

- Wely fractional integral:

$$(W^{-\gamma}f)(x) = \frac{1}{\Gamma(\gamma)}\int_{x}^{\infty}(t-x)^{\gamma-1}f(t)\,dt, \quad Re(\gamma)>0$$

- Grünwald-Letnikov fractional derivative [2]:

$$D^{\alpha}f(x) = \lim_{h\to 0}\frac{1}{h^{\alpha}}\sum_{k=0}^{\infty}(-1)^k\frac{\Gamma(\alpha+1)f(x-kh)}{\Gamma(k+1)\Gamma(\alpha-k+1)}$$

  where $\Gamma(\cdot)$ is the Gamma function

Significance in Present World

It has become a vibrant topic due to its excellent applications and amazing real-life results.

Image processing

Sobel and Canny proposed edge-detecting methods in 2018 [1]. These methods effectively detect sharp edges, but they may struggle with images containing complex textures. Fractional equations help in textured or fractal-structured photos. It also performs very well in blurry and low-quality images.

Behavior of viscoelastic material

Maxwell and Kelvin-Voigit's models for viscoelastic material behavior were based on integer order derivatives. Fractional calculus gives more accuracy to the effects of its behavior, as it has a memory effect. When we use fractional derivatives in stress-strain relation, it shows that viscoelastic material responses depend not only on current strain but also on past strain.

Control theory

Fractional controllers, such as fractional PID controllers modified controllers since their inception. Using fractional derivatives improves the performance of drones, robots, and all dynamic systems dependent on the. Moreover, fractional derivatives enhance the performance of drones, robots, and other dynamic systems that depend on the fractional memory effect.

Electrical engineering

Fractional differential equation is widely used in the modeling of circuits. It helps in the study of capacitors, inductors, etc. Fractional calculus can describe the system's behavior more effectively than standard methods.

Biology

Fractional models are gradually used in biological systems, like anomalous diffusion [3], neutral network, pharmacokinetics, drug release models [4], etc.

Finance

Economics models are based on long memory processes. The price of stocks in real life depends on historical data. Fractional models can capture the prolonged effects of shocks or fluctuations in the market.

Quantum systems

The fractional Schrödinger equation is widely used in quantum dynamics systems. Fractional calculus can model the evolution of quantum particles in complex, disordered environments where traditional models fall short. It also provides deeper insights into quantum tunneling.

Conclusion

Fractional differential equations offer a powerful tool for all systems with memory effects. The fractional differential equation became an extraordinary differential equation. It continuously influences human knowledge and progress across a wide range of advancing research, technology, and daily living.

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Multi-Objective Optimization: Nature, Applications, and Solution Techniques

Ekata Jain

Multi-objective optimization, or MOO, is a branch of mathematics that handles problems characterized by two or more opposing goals. MOO, as opposed to single-objective optimization, which always aims to find one optimal solution to a problem, tries its best to accomplish two or more goals simultaneously. This conflict among objectives makes MOO complex but necessary for scenarios where a decision must be made with multiple criteria. For example, designing a vehicle involves maximizing safety, minimizing cost, and maximizing fuel efficiency. These goals are usually conflicting; adding safety features drives up costs and cuts fuel economy. So, an equilibrium between them has to be met. The essence of MOO is making compromises on the targets to achieve the final set of optimal solutions for decision-makers to choose from, depending on the adopted preference.

Multi-objective optimization has numerous real-world applications that require trade-offs between competing objectives. In engineering design, for example, it is used to improve a product's performance, cost, and safety features. Engineers in the automotive and aerospace industries strive to achieve maximum performance while controlling costs and maintaining safety standards. In supply chain management, companies seek to optimize costs, delivery time, and environmental impact to enhance operational effectiveness and sustainability. In finance, MOO aids in the optimization of portfolios through diversification as it helps balance risk and return on investment. In medicine, it is helpful in treatment planning to achieve maximum effectiveness with minimal side effects while controlling costs for the healthcare provider and the patient. Likewise, in energy systems, MOO is used to balance the level of power generated with the operating cost and the negative effect on the environment to promote sustainable energy practices. All these examples relate to each other as they demonstrate MOO as one of the most critical aspects of solving numerous interrelated problems concerning diverse fields of knowledge.

MOO problems are defined by having coexisting conflicting objectives and, therefore, a trade-off of optimal set solutions instead of an optimal solution. This brings us to the notion of Pareto Optimality, where a solution is said to be Pareto optimal if no other solution can be offered that improves one objective without worsening at least one other objective. The complete collection of Pareto optimal solutions establishes the so-called Pareto front, illustrating the trade-off between the conflicting objectives. Within the identification of the Pareto set, there still lie possibilities to choose from, and the decision maker must choose from the Pareto front, representing the solutions with the utmost importance to them. This gives rise to the concept of multi-criteria decision-making (MCDM) that helps rank or choose from Pareto optimal solutions by introducing the decision maker's choice. Thus, MOO problems not only require mathematical optimization but also empower represented decision-making to select the most appropriate compromise solution. A few classical methods have been developed for MOO problems. One of the classical methods is the Weighted Sum Method, which is the simplest to implement, where all objectives are weighted to form a single objective. This approach could prove ineffective for finding Pareto optimal solutions when the Pareto front is non-convex. The ϵ-Constraint Method narrows the search space by optimizing one objective while transforming others into constraints with predetermined limitations. Goal programming is appropriate for issues where particular targets are desired because it establishes ambition levels for each aim and reduces departures. A utility function that reflects the decision-maker's preferences aggregates various objectives in a different method known as a utility function. Although these traditional approaches work well, they frequently have drawbacks when handling complicated, non-linear, and multi-modal problems.

Limitations of classical techniques have prompted the development of new techniques and approaches. The newer population-based methods, for example, Non-dominated Sorting Genetic Algorithm (NSGA) II & III and Strength Pareto Evolutionary Algorithm (SPEA) 2), have been very successful in solving most MOO problems since they can effectively search the problem space by simultaneously considering different Pareto optimal solutions. These methods mimic the natural selection procedure and apply operators like selection, crossover, and mutation to develop solutions over generations. Another modern approach is Multi-Objective Particle Swarm Optimization (MOPSO), which considers social behavior in flocks of birds for exploring the search space efficiently by Updating the positions of particles (solutions) in terms of their own experiences and those in the neighborhood of other particles. Hybrid algorithms take advantage of evolutionary techniques and local searches to achieve better accuracy in convergence. With the rise of machine learning, interest in integrating Machine Learning Models as surrogate models to estimate complex objective functions and reduce computational expenditure is gaining momentum.

Though much advancement has been made in MOO, scalability and computational complexity remain significant challenges. A larger number of objectives and constraints exponentially increase the search space, requiring even more computational resources and efficient algorithms. Adding a decision-maker's preferences effectively constitutes another challenge because preferences can change dynamically during decision-making. Adaptive algorithms may be necessary to address changing objectives and constraints in dynamic, uncertain environments. In conclusion, multi-objective optimization continues to be a crucial technique for resolving conflicting objectives in intricate situations as the need for sophisticated decision-making grows. Its adaptability and efficiency have proven helpful in various domains, from engineering and banking to healthcare and energy systems. By examining trade-offs and providing various ideal options, MOO enables decision-makers to make well-informed and well-rounded decisions. Without a doubt, its ongoing advancement and use will lead to more practical and efficient answers to a range of real-world problems.

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Importance of Operations Research

Gannamaneni Sai Charan

Operations Research (OR) has become essential for efficiently managing day-to-day activities in the modern world. It has attracted many researchers to work in various fields, from optimization to queueing theory and stochastic modeling. It is adopted to make better decisions that provide the best outcome efficiently. Over the years, researchers have focused on developing efficient models for a better understanding of systems of interest that offer better insights leading to strategic decision-making, which is a key concept in the studies of OR. Many theories have been developed focusing on the progress of OR, and it has wide-ranging applications in healthcare, telecommunication, transportation, resource management, military operations, logistics, etc.

While studying OR, the first step is the challenge of identifying and mathematically constructing the problem. The mathematical construction may be linear or non-linear, depicting the real-life system in a well-constructed manner. The next question is ". How do we solve the mathematical construction?" Numerous analytical methods have been devised to solve various systems, yielding the desired results. The findings are applied to offer guidance for improved decision-making. Finally, the optimal option found during decision-making is implemented to achieve more effective outcomes.

Numerous important topics are studied in OR, such as queueing theory, stochastic modeling, optimization, supply chain management, game theory, simulation modeling, etc. These theories offer the best solutions and aid in the comprehensive understanding of complex situations. Every OR topic has significance and applicability across a wide range of industries. The queueing theory originated in developing efficient telecommunication management systems by reducing a customer's wait time while all the lines are busy. This has led to the development of intriguing queueing models that find use in various domains, including computer systems, manufacturing systems, network management, healthcare management, etc. Lately, advancements in healthcare queueing management have assisted in allocating the available resources optimally while meeting healthcare demands. Furthermore, studies on optimally distributing resources have great importance, especially with fast-paced work in inventory management. Another important concept of OR is optimization, which has vast applications. Optimization tools are adopted to optimize the effective usage of resources while satisfying the imposed restrictions. The main objectives considered by organizations adopting optimization techniques are to reduce cost, increase profit, and minimize environmental damage and risk. Using the abovementioned studies, many OR tools have been developed that are very advantageous and adopted by many industrial, healthcare, and telecommunication organizations.

In conclusion, OR is a fascinating area of study, and the advancement of effective systems heavily depends on its development. Given the broad spectrum of OR difficulties, OR tools can play a crucial role in expanding businesses, sectors, and the healthcare industry, all of which contribute to the development of governments and nations.

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The Ethics of Mathematical Machines

Rajarapu Mahesh

The Machines Are Coming...With Calculators!

Think about this? You are sitting at the desk working hard on an important math-problem-maybe something involving prime numbers, which are odd creatures, as they cannot be divided by other numbers except one and themselves.You are focused, writing carefully, when suddenly a neural network, a special computer program, arrives and it calmly says, "I have already solved it for you. Here is your answer!."

What happens if machines solve math problems before we do? This is not just an idea from stories, it is a serious question about math, thought, and our role as humans. Neural networks are advanced programs that use a helpful math method called gradient descent to find answers quickly, much faster than a person might. But if they begin to solve our cherished math problems, the ones we love to explore what does this mean for us? Let us look into this together, with respect and a deep appreciation for the journey ahead.

The Math That Makes It Work: Understanding Gradient Descent

To initiate, let's introduce a new term, the **gradient descent**. This elegant trick from the field of mathematics is generally used for optimizing neural network parameters. Let us assume you have something like a ball, and you want to drop it down the hill to reach ground level. Neural networks also function in the same way, incrementally adjusting some parameters called *weights* to rectify and minimize errors. Thus, mathematically, we get

$$w_{new} = w_{old} - \eta \cdot \frac{\partial E}{\partial w} \tag{11.1}$$

We'd like to interpret it informally.

- w: weight, like a number the network uses to decide.
- Before performing any actions: w_{old} is the weight.
- Once it is updated, the weight becomes $wnew$.
- Learning rate, which is also called eta, tells how big and small every step is. This means how much the ball will be pushed to roll with each step: η.
- The gradient or slope of the hill shows $-\frac{\partial E}{\partial w}$ the steepness at which the ball will roll to the bottom.
- The error E: how wrong the network's guess was. Therefore, thatś the number we want to minimize.

In other words, the network starts by taking a weight, inspects the slope of the error, steps down the error, and repeats until it finds the lowest point of error. This is the ball rolling until it settles at the bottom and holds its position.

This showed the working method of proof in Mathematics. Say you want to prove something: that the sum of the first n odd numbers equals n^2 (n squared). For example:

- For 1 odd number: $1 = 1^2 = 1$.
- For 2 odd numbers: $1 + 3 = 4 = 2^2$.
- For 3 odd numbers: $1 + 3 + 5 = 9 = 3^2$.

Essentially, someone would prove this using induction. Could you start with a small number like one and see if the assertion is valid? The next step would be to assume induction to hold for some number k (i.e., the sum is k^2) and then proceed to check for the following number $k+1$ by augmenting the following odd number, which happens to be $2k+1$. The math here would look like this:

$$S_k + (2k+1) = k^2 + 2k + 1 = (k+1)^2 \tag{11.2}$$

Here, S_k: up to k, is assumed to be k^2. Adding $2k+1$ gives $k^2 + 2k + 1 = (k+1)^2$. This shows that this declaration has to be true for every number, stage by stage.

But a neural network does it differently. Instead of using steps like induction, it looks at many examples and data and figures out the pattern. After seeing many sums, it might just say, ". The answer is n^2." **Machine Method:** A machine does it differently. It looks at many examples, like billions of sums, and learns the pattern from them. After studying the data, it says the answer is n^2. It also says its guess is almost perfect, with only a tiny error, like 0.0001%. The machine doesn't explain the steps; it just answers quickly based on what it has learned.

Humans take time to prove things carefully and show every part. Machines use lots of data to find the answer fast, but they don't show how they got it. Both find the correct answer, but they work very differently. So, gradient descent helps neural networks find answers by making small changes until they're right. They can even tackle proofs, but they do it in their way fast and direct, not slow and careful like humans. Both ways work, but they show how machines and people think differently about math.

The Big Question: Machines and Mathematicians

So, **what happens if machines solve math proofs before we do?** Let's look at this critical question carefully and step by step.

Part 1: Machines Are Very Fast

Machines are rapid in operation. They are solving problems at a speed much faster than humans. In 2019, some smart people were using a computer program to shorten and improve mathematics proofs, very much in the way of offering graph theory.[4] Noreena Hertz wrote about this in *Nature* in 2021 in an article titled "AI tackles tough math problems."[2, 4] They did not finish any big problems like Fermat's Last Theorem but eased the load of human labor. Would we cheer for machines if they solved a major mathematical problem such as the Riemann Hypothesis before us and we felt pity for ourselves? Worth pondering.

Part 2: Proofs Are Special to Humans

Mathematical proofs aren't just solutions. They are beautiful ideas we embed into our creations. Long ago, a mathematician by the name of Euclid proved that there are always more prime numbers no matter how many you find[3]. He would say, "Assume that there is a greatest prime number, call it p. Multiply all the primes up to p and add 1."

Examples have gone thus:

$$N = 2 \cdot 3 \cdot 5 \cdot \ldots \cdot p + 1 \tag{11.3}$$

This new number, N, is either a prime itself or can be divided by a prime bigger than p. This means there's always a bigger prime, so there are infinite primes. It's a clear and lovely way to show the idea.

But a machine might just look at numbers and say, "Yes, there are infinite primes," without showing the steps. It doesn't care about making it beautiful. If machines start doing all our proofs, will we lose this special human touch? Or will they help us think of even bigger ideas? We need to consider both sides.

Part 3: What Happens If Machines Do Everything?

Imagine asking a machine, "Can you prove Goldbach's Conjecture?" This conjecture says every even number bigger than 2 can be made by adding two prime numbers. A machine might answer, "Yes, it's true," and stop there.

If machines solve all our math problems, what will we do? Will we have no work left, just watching machines get all the praise? Or they could help us by giving us answers so we can try more challenging problems ourselves. For example, if a machine solves something big, we might say, "That's amazing! Now, let's find something even more exciting to work on." It could push us to do better.

The Upside: Machines as Helpers

Here's a positive idea. Neural networks can be our helpers, not our bosses. In 2022, AlphaCode from DeepMind's program worked on coding problems with some math ideas in them[5]. This was written about in a magazine called *Science*, in an article named "AI steps into pure math," The machine didn't take over people; it helped them by making things faster and easier. For example, imagine a machine noticing a pattern in the gaps between prime numbers. Then, humans use that pattern to make clear and excellent proof. When we work together, we can do great things.

So, machines don't have to be our competitors. They can be like guides. They might say, "Here's the answer," and then we take it and make it better or more beautiful. Here's why humans are special and what we can do:

- Humans are unique because we think deeply, create new ideas, and explain things in easy-to-understand and enjoyable ways.
- Machines can find answers quickly using lots of data, but they don't tell us the story behind the answer. Humans add that story.
- If machines do all the hard work, like finding patterns or solving big problems—we can focus on making new questions, finding meaning, and sharing the beauty of math with others.

In this way, machines do challenging tasks, and we keep up the critical task of thinking and creating. Both sides win machines help us, and we use our unique skills to make math even better.

Conclusion: A Bright Future with Machines

What does it mean for us mathematicians if neural networks solve math theorems? It's an exciting thought. Sometimes, we might worry because machines are so good and fast, using tools like **gradient descent** to find answers quickly. But there's also wonder they can do amazing things! And best of all, I hope they can help us with our work.

Humans have something special: we think creatively that machines can't copy yet. When you work on a math problem next time, you might wonder, "Can a machine do this faster?" That's okay; keep working on it yourself. Even if machines get good at proofs, we will always have our unique way of understanding and exploring math. With machines as our helpers, we can make math even more remarkable for everyone.

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The Legacy of Emmy Noether in Ring Theory

Theophilus Gera

The title “Mother of Abstract Algebra,” given by Irving Kaplansky (1917-2006), correctly stands for Emmy Noether (1882-1935). Her pioneering work stood the test of time and continues to inspire algebraists. Her work [6] (a translation is available at [1]) seems understandable and quite often negligible to us as compared to recent advances in mathematical studies. Still, it was a turning point of restructuring perspectives introduced and studied by J. E. M. Wedderburn (1882-1948) and their descendants. It is right to say that she was the reason for the “advent” of modern algebra.

In her paper ([6]), R. Dedekind’s[1] theorem of the finite chain was put forward in an ideal theoretic approach, which ruled out the “axiom of choice.” In fact, Lasker’s[2] theory on the decomposition of primary ideals led Noether to prove it in a general context. In the same paper, she gave a counter-example stating that a commutative case is not superfluous (a detailed study is found in [7]).

In ring theory, Noether’s work influenced two central theorems, i.e., Cohen’s[3] Theorem [2] and Kaplansky’s Theorem [3]. The lack of a unified approach pushed for a more significant unifying theory. Lam and Reyes [4] [5] [8] [10] published a series of papers in this direction by conceptualizing Oka and Ako ideal families and studying the “prime ideal principal” (which is noncommutative analog of “maximal ideal implies prime” concepts). Reyes further extended the theorems in noncommutative settings in [9].

As observed earlier, a small part of Noether’s work developed literature, which is hard to track. It is even hard to describe the impact Noether created within a century of her pioneering work, and it becomes even more complex and challenging to record the entire literature in the near future. The flavor of ideal theory and the legacy of Emmy Noether is hard to explain as her theory has been inherited in almost every part of Algebras and beyond. The Conference on 100 Years of Noetherian Rings, organized by IAS, Princeton, in 2023, is a testimony and witness to her work. The future of algebra is Noetherianism to its core.

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The Importance of Mathematics and Its Applications in Performance Analysis of a Device Using SCAPS-1D

Sodari Saiminnu

In order to comprehend and optimize semiconductor devices, mathematics is essential. It offers crucial tools for simulating recombination processes, electric fields, and charge carrier dynamics—all of which have an impact on device performance. Numerical simulations are frequently used in semiconductor physics to assess and improve device efficiency. SCAPS-1D (Solar Cell Capacitance Simulator – 1D) is one such potent simulation tool.

In semiconductor research, SCAPS-1D is commonly used to model and analyze electrical properties, especially in the study of solar cells. It uses mathematical formulas to resolve intricate issues pertaining to efficiency, recombination, and charge transfer. The significance of mathematics in performance analysis with SCAPS-1D and its role in semiconductor device optimization are examined in this paper.

Mathematical Foundations of SCAPS-1D

Basic mathematical formulas that explain charge carrier behavior, electric fields, and recombination processes control how semiconductor devices operate. Among the fundamental mathematical ideas used in SCAPS-1D simulations are:

Poisson's Equation

Poisson's equation describes the relationship between the electrostatic potential and charge density in a semiconductor:

$$\nabla.(\epsilon \nabla V) = -\rho$$

Continuity Equations

Charge carrier transport is modeled using continuity equations, which account for generation, recombination, and movement of electrons and holes.

Numerical Methods in SCAPS-1D

SCAPS-1D employs mathematical techniques to solve semiconductor equations:

1. **Finite Difference Method (FDM)**: Discretizes equations for numerical approximation.
2. **Newton-Raphson Method**: Solves nonlinear equations efficiently.
3. **Matrix Algebra**: Used to solve large systems of equations related to device behavior.

Performance Analysis of Devices Using SCAPS-1D

Mathematics helps extract key performance parameters from SCAPS-1D simulations:

Current-Voltage (J-V) Characteristics

The J-V curve provides:

- **Short-circuit current** – Carrier generation efficiency.
- **Open-circuit voltage** – Determines recombination effects.
- **Fill Factor (FF)** – A measure of efficiency.

Capacitance-Voltage (C-V) Analysis

Capacitance is calculated using: $C = \frac{dQ}{dV}$

Bandgap Engineering and Energy Levels

Energy band diagrams are modeled mathematically to examine the effects of bandgap changes on charge transport and recombination. High-efficiency device design is aided by the optimization of band alignment using mathematical models.

Thickness Optimization for Improved Efficiency

SCAPS-1D uses optimization and numerical differentiation techniques to identify the optimal thickness of each layer in a semiconductor device to attain maximum efficiency.

Temperature Dependence and Thermal Effects

The performance of devices under various environmental circumstances can be predicted with the use of mathematical models of temperature-dependent carrier mobility and recombination rates.

Interface and Defect Modeling

The effects of bulk and interface flaws on carrier transport can be simulated with the aid of mathematical models.

Optical Properties and Light Absorption Analysis

SCAPS-1D also simulates light absorption and its impact on device efficiency.

1. **Absorption Coefficient and Optical Generation**: The optical absorption spectrum is analyzed to study how different wavelengths contribute to charge generation. The impact of layer thickness and bandgap tuning on light absorption is evaluated.

2. **Quantum Efficiency (QE) Analysis**: The External Quantum Efficiency (EQE) and Internal Quantum Efficiency (IQE) curves provide insights into photon absorption and carrier extraction.

3. **Impact of Anti Reflection Coatings (ARC)**: SCAPS-1D can model ARC layers to reduce reflection losses and enhance light absorption in solar cells.

Conclusion

In semiconductor research, mathematics is an essential instrument that sheds light on charge transport, recombination, and device performance. Using numerical simulations and mathematical modeling, SCAPS-1D analyzes and optimizes semiconductor devices, especially solar cells. Researchers can improve efficiency and create next-generation semiconductor devices by resolving complicated differential equations and deriving performance metrics. Anyone in semiconductor physics must understand these mathematical concepts since they improve device design and optimization. The simulation tool SCAPS-1D is essential for conducting thorough performance analysis and optimizing semiconductor devices, especially thin-film and multilayer solar cells. Its numerical method provides a comprehensive grasp of charge carrier dynamics, recombination mechanisms, energy band topologies, and the impact of material parameters on device performance.

The ability of SCAPS-1D to simulate various device architectures and material compositions is one of its most significant benefits since it enables researchers to investigate novel technologies without immediately investing in costly fabrication and testing. SCAPS-1D helps alter factors, including doping concentration, layer thickness, defect densities, and interface conditions, to find the best configurations that optimize efficiency and stability.

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